

45 minutes

Purpose of this lesson

This lesson is designed to be incorporated into Module 1, core lesson 4, in which students learn about potential divider circuits. In this lesson, students will:

- Practice solving simultaneous equations.
- See how simultaneous equations are a useful tool for solving real world problems.
- Prove the potential divider equation used in Module 1, core lesson 4.

Analyzing electric circuits

Note to teacher: There are several ways to approach this which are all in the end equivalent. All approaches use Ohms Law ($V=IR$) to write down two simultaneous equations which are solved for the two unknown quantities V_{out} and I (voltage measured across the $10k\Omega$ resistor and current flowing round the circuit). Two approaches are included below. Students may attempt problems using either approach or both.

Electricity involves electrons flowing through the wires and components of a circuit. We can make analogy with water flowing through a system of pipes (see lesson 2 – Introduction to electricity). Electric current (denoted I) is equivalent to the amount of water flowing through a pipe (e.g. volume of water per second). Voltage (denoted V) is equivalent to the pressure which forces the water through the pipe.

Look at the simplified circuit diagram, fig. 1. Clearly any current which passes through the $10k\Omega$ resistor must also pass through the thermistor (think about

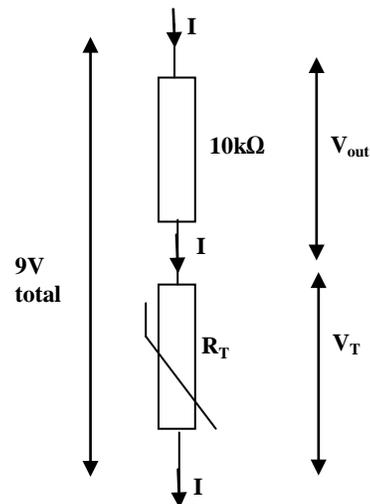


Fig. 1. Thermistor (bottom) and resistor (top) form a *potential divider* circuit. Due to the battery, there is 9V across the two resistors. This total 9V is *divided* between the resistor and the thermistor. Some of it, V_{out} , is applied across the resistor and the remainder V_T is applied across the thermistor.

water flowing through two pipes connected end-to-end).

Note that:

- The same current, I , passes through both resistors.
- The total voltage across both resistors is 9V due to the battery.
- This voltage is divided between the $2K\Omega$ resistor and the thermistor, whose resistance, R_T , varies with temperature.
- Using a multimeter, we can measure the portion of the voltage across the $2K\Omega$ resistor. This will be our sensor's "output signal", so we have labeled it V_{out} .
- Note $10k\Omega$ means "ten thousand Ohms".

This sensor circuit is designed to measure temperature. Therefore, to analyze our sensor, we want to derive a formula which relates the sensor's output signal, V_{out} , to the quantity which varies with temperature, R_T .

We can analyze this circuit using Ohm's law, which states that for any resistor, R , the voltage, V , and current, I , are related by the equation $V=IR$. We will apply Ohm's law twice to form two simultaneous equations.

Approach 1

1) The total battery voltage (9V) acts across the combination of two resistors (the resistances add) causing an unknown current, I , to flow through both.

$$\begin{array}{l} \text{Hence, Ohm's law} \quad V \quad = \quad I \quad \times \quad R \\ \text{Gives} \quad \quad \quad 9 \quad = \quad I \quad \times \quad (10000 + R_T) \quad \quad \quad \text{equation 1} \end{array}$$

2) The measured voltage, V_{out} , acts across the $10k\Omega$ resistor alone, causing the same current, I , to flow through it.

$$\begin{array}{l} \text{Hence} \quad \quad V \quad = \quad I \quad \times \quad R \\ \text{Gives} \quad \quad V_{out} \quad = \quad I \quad \times \quad 10000 \quad \quad \quad \text{equation 2} \end{array}$$

i) Look at the temperature-resistance data that you collected in *Lesson 3 – Temperature vs. resistance characteristics of a thermistor*. For each temperature that you experimented with, substitute the value of thermistor resistance, R_T , into equation 1

above. Now solve the pair of simultaneous equations (equation 1 and equation 2) to find the corresponding value of output voltage, V_{out} . Hint, use the method of substitution or another method to eliminate I .

Temperature / °C	0	10	20	30	40	50	60	etc
R_T (look up from Lesson 3)								
V_{out} (find by solving equations)								

ii) Plot a graph of Temperature versus V_{out} , with temperature on the vertical axis (y-axis) and V_{out} on the horizontal axis (x-axis). Compare this with similar data that you have measured or calculated in lessons 4 and 5.

More advanced:

iii) In part i) you solved the pair of simultaneous equations many times, using a different value for R_T each time, to find values of V_{out} . Now see if you can find a general formula for V_{out} in terms of R_T . Hint, keep R_T as a letter, but treat it the same way you did when you assigned it numerical values in part i) above.

Approach 2

Look at figure 1. The total voltage of 9Volts (from the battery) must be equal to the voltage across the thermistor plus the voltage across the 10k Ω resistor (we can call these V_T and V_{out}).

i.e. $V_T + V_{out} = 9$
 rearrange to get $V_T = 9 - V_{out}$

Now we can apply Ohms law to the thermistor:

Ohms law $V = I \times R$
 Gives $V_T = I \times R_T$

Now combine the above two equations, by eliminating V_T (e.g. by method of substitution) to get:

$$(9 - V_{out}) = I \times R_T \quad \text{equation 1}$$

Now apply Ohms law to the 10kΩ resistor:

$$\text{Ohms law } V = I \times R$$

$$\text{Gives } V_{out} = I \times 10000 \quad \text{equation 2}$$

i) Look at the temperature-resistance data that you collected in *Lesson 3 – Temperature vs. resistance characteristics of a thermistor*. For each temperature that you experimented with, substitute the value of thermistor resistance, R_T , into equation 1 above. Now solve the pair of simultaneous equations (equation 1 and equation 2) to find the corresponding value of output voltage, V_{out} . Hint, use the method of substitution or another method to eliminate I .

Temperature / °C	0	10	20	30	40	50	60	etc
R_T (look up from Lesson 3)								
V_{out} (find by solving equations)								

ii) Plot a graph of Temperature versus V_{out} , with temperature on the vertical axis (y-axis) and V_{out} on the horizontal axis (x-axis). Compare this with similar data that you have measured or calculated in lessons 4 and 5.

Optional:

iii) In part i) you solved the pair of simultaneous equations many times, using a different value for R_T each time, to find values of V_{out} . Now see if you can find a general formula for V_{out} in terms of R_T . Hint, keep R_T as a letter, but treat it the same way you did when you assigned it numerical values in part i) above.

Answers – example analysis

From approach 1 above:

Part i) For example, at $T = 0$, $R_T = 27200\Omega$. Substitute these values into equations 1 and 3:

$$9 = I \times (10000 + 27200) \quad \text{equation 1}$$

$$V_{out} = I \times 10000 \quad \text{equation 2}$$

From eqn 1: $I = 9/(10000 + 27200) = 0.000242$

Substitute into eqn. 2, gives: $V_{out} = 0.000242 \times 10000$

Hence $V_{out} = 2.42V$

Part iii)

We can solve this pair of simultaneous equations using substitution, eliminating **I** and leaving us with a useful formula relating V_{out} to R_T .

Rearrange equation 1 to give:
$$I = \frac{9}{(10000 + R_T)}$$

Now substitute this expression for **I** into equation 2, giving:

$$V_{out} = \frac{9}{(10000 + R_T)} \times 10000$$

$$\text{Hence } V_{out} = \frac{90000}{(10000 + R_T)} \quad \text{equation 3 (final formula)}$$

V_{out} is the output of your sensor system. R_T is the resistance of your thermistor, which varies with temperature. Temperature is the quantity you are trying to measure with your sensor.

Problem for more advanced students with knowledge of logarithms

In Lesson 3 – Temperature vs. resistance characteristics of a thermistor, you observed that the resistance of your thermistor decreases with temperature according to a curved relationship.

- i) What kind of relationship is this?
- ii) Find an equation or formula that mathematically describes this decrease in resistance with temperature.
- iii) Use the data you gathered in Lesson 3 to find values for any constants in your equation.

iv) Substitute this formula into equation 3 above, to find a mathematical expression which relates the output of your temperature sensor (V_{out}) to temperature, T .

Answers:

i) Exponential decay.

ii) $R_T = Ae^{-kT}$

iii)

Temperature °C	0	80
Resistance KΩ	27.2	1.7

Hence:

$$27.2 = Ae^{-k \times 0} = Ae^0 = A \times 1 = A$$

i.e. $A = 27.2$

$$\text{And } 1.7 = Ae^{-k \times 80} = 27.2e^{-k \times 80}$$

$$\text{Therefore } e^{-k \times 80} = 1.7/27.2 = 0.0625$$

Take logs of both sides:

$$\ln e^{-k \times 80} = \ln 0.0625$$

$$\text{Therefore } -80k \ln e = \ln 0.0625$$

$$\text{Therefore } -80k = \ln 0.0625$$

$$\text{Therefore } k = 0.0347$$

$$\text{iv) } V_{out} = \frac{90000}{(10000 + R_T)} = \frac{90000}{(10000 + Ae^{-kT})} = \frac{90000}{(10000 + 27.2e^{-0.0347T})}$$